Foundations for success in mathematical competitions: A study of best praxis in lower secondary schools in Norway

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Abstract: The purpose of this study is to investigate the following questions: What factors leads to success in mathematics, and how can these success factors and qualities be described? Will the teacher's education and pedagogical praxis have an impact on good learning results? We report results from a case-control association study on among high achievement classes in mathematics in Norway. The data were collected from matched pairs of schools, paired on the basis of location and socioeconomic status. The questionnaire was first distributed to teachers in 38 Norwegian secondary schools at grade 9, which have had repeated success in the annual KappAbel competition in mathematics. Subsequently, 38 teachers at schools without success were contacted, and answered the same questionnaire. The main findings of the study are the following: The formal academic competence of the teacher is the best predictor for good results. Moreover, the pedagogical profile is reason oriented, where students are challenged to evaluate and substantiate their arguments, and spreadsheet is used for exploration and computation.

Keywords: KappAbel; Teacher competence; Pedagogical profiling

1. Introduction

The teacher

Hanushek and Rivkin (2006) has given an overview of international research literature which shows that there is a big difference between teachers with regard to what effect they have on students learning. However, little is known from existing high-quality research about what effective teachers *do* to generate greater gains in student learning (National Mathematics Advisory Panel 2008, p. xxi). What active ingredients characterize a good teacher, is still a question with no clear answer, and the research literature conclude that it is difficult to

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associate the quality differences to the objective characteristics of the teachers. Some new research (Clotfelter, Ladd and Vigdor, 2010), however, have found significant and positive effects on learning results related to teachers' professional skills and their *site* of education. Further research is needed to identify and more carefully define the skills and practices underlying the differences in teachers' effectiveness.

In mathematics there has been an extensive discussion about the significance of teacher subject matter knowledge and students success (Hill & Ball, 2005). But even if there is an agreement that mathematical content knowledge is a precondition to be able to teach mathematics, studies that examined the influences of teachers' subject matter knowledge on student result have produced mixed findings (Hattie 2009). Ahn and Choi (2004) conducted a meta-analysis based on 27 primary studies of mathematics achievements in order to examine the relationship between teachers' subject matter knowledge and student learning. They found a very low effect size between knowing mathematics and student outcomes (effect size d=0.12). These results suggest that subject matter knowledge, as currently transmitted to teachers-in-training by colleges of education, is not very useful in the elementary school classroom. It may also be argued that it is probable that subject matter knowledge do have a positive impact on teaching up to some level of basic competence, but less so after that (Hattie, 2009; Monk, 1994).

On the other hand, it is well documented in the research literature an effect of teacher verbal and cognitive ability on student achievement. Every study that has included a valid measure of teacher verbal or cognitive ability has found that it accounts for more variance in student achievement than any other measured characteristic of teachers (Greenwald, Hedges and Lane, 1996; Ferguson and Ladd, 1996; Kain and Singleton, 1996). Greenwald, Hedges and Laine even point out that the rational ability of the teacher may be more powerful than teacher training.

Concerning the teacher's education within problem solving, Hattie underline (2009, p. 210):

> The teacher characteristic with the most positive effect on student's performance was specialist training in heuristic methods (effect size d=0.71). These methods include, for example, Pólya's (1945) four phases of: (1) understanding the problem, (2) obtain a plan of the solution, (3) carry out the plan, and (4) examine the solution obtained.

Therefore, teachers' educational level and skills are included in this study based on an assumption that this may affect the academic priorities and methodological choices. Some previous studies show that teachers' educational background may be important for students' academic achievement, and we need to know more on how this is linked with teachers' praxis theories and teaching.

The pedagogy

The general pedagogical praxis orientation may be categorized in different ways. Hattie (2009) uses the concepts of teacher as *activator* and the teacher as *facilitator*, where the terms stand for different roles in the management of education. The teacher as "facilitator" is more facilitating the activities, in contrast to the teacher who actively participates directly in it to convey an educational content. Lie et al. (1997) refers to a similar analysis of the teaching practices in science and mathematics by two different models, "*Teaching 1* and *Teaching 2*". In the Norwegian section of SITES study (Ottestad 2008), funded by the Norwegian Ministry of Education and Research, a more normative term is applied, such as "the traditionally oriented teacher" in contrast to "teacher oriented towards lifelong learning".

	Praxis Description I	**Praxis Description II**
Lie, 1997	**Teaching 1:** is working with the project methods, group work, use of ICT	**Teaching 2:** Traditionally, teacher-controlled teaching methods
Ottestad, 2008	**Lifelong learning:** Group work. Cooperative learning and problem-based learning. Students have an active role in identifying issues, as well as the way one should solve the tasks. The teacher typically takes the role of launch pad in the learning processes	**Traditional orientation:** orientation toward knowledge and achievement as measured by traditional means (tests, exams). The teacher typically takes the role of instructor and evaluator. Students follow instructions and work with assigned tasks.
Hattie, 2009	**Facilitator:** Problem-based learning, project methods, the Internet supported learning, computer games and simulations.	**Activator:** Teacher-directed teaching methods. The teacher actively participates in teaching, giving direct instructions on effort, learning and behavior.

Table 1

These three ways to describe different teaching practices shows several similarities as summarized in Table 1, and reminds us of the well-known division of the teacher-centered versus student-centered teaching. However, this is an over-simplified subdivision, since teacher-controlled teaching can be dialogic and student active learning can be authoritarian

The various praxis theories have been associated with different learning results. Lie et al. (1997, p.203) describes the "Teaching 1" seems clearly negative impact on mathematics achievement, while instruction 2, traditional teacher-controlled education, is clearly the best outcome:

> It is for us a paradox that the ways of working which is highly recommended for the time, project work, group work and use of IT, appears to be linked to the weak performance in mathematics.

Hattie (2009) also emphasizes the teacher's major influence on students' learning results. This applies in cases where the teacher actively participates in teaching, giving direct instructions on effort, learning and behavior. Nordahl (2005) points out that this is consistent with the conclusions of the PISA reports, where it is expressed that the somewhat weaker results in the Norwegian school system can be linked to the teachers too much has been supervisors and in the little stand forth as leaders. It is further emphasized that the student activation has been more important than structural and technical requirements related to learning. This means that the practice description (II) listed in the right column of Table 1, achieve the best learning results. In the report *"Time for heavy lifting"* (Kjærnsli et al., 2007) states:

> There is a clear tendency for a strong emphasis on students' exploration of ideas is linked to low achievement. This message is fairly clear, although these results for

> many may come unexpectedly. These results imply at least a powerful provocation for those who have argued for the importance of as many as possible "degrees of freedom" in practical work.

In the SITES – study (Law, 2008; Ottestad, 2008), however, researchers believe that the pedagogical orientation of schools and education authorities should work towards greater degrees of freedom. Under the heading *"Education for the 21st century,"* teachers should mainly be facilitators, while students have an active role in efforts to identify interesting problems and select methods to resolve these. It is reported that the learning results of these methods provide a relatively small but positive increase in student's inquiry skills (including information-handling, problem-solving and self-directed learning skills) and the ability to cooperate. But is unclear what kind of knowledge in mathematics and science this gives, which are the subjects this survey reviews.

The more specific pedagogy theory related to mathematics is traditionally categorized explicitly as *reason*-oriented versus *rule*-oriented approaches. People are in general used to speak about meaningful learning, and Skemp (1976) introduced the well known discern between *relational* understanding versus *instrumental* understanding. This issue has later been considered by means of different terminologies in the literature: *conceptual* knowledge versus *procedural* knowledge (Hiebert, 1986), *analytical* thought processes versus *pseudo-analytical* thought processes (Vinner, 1997) and *creative* reasoning versus *imitative* reasoning (Lithner, 2008). It will be of great interest to look for differences in learning outcome between these two kinds of pedagogical approaches.

The technology

In mathematics, Norwegian teachers are encouraged to use Internet and a variety of software: dynamic geometry and symbolic calculation software, spreadsheet, etc. Most schools and teachers are still characterized by patchy uncoordinated provision and use, and for the time being the impact by IT on learning outcome in mathematics appears to be unclear and contradictory (Balanskat, Blamire and Kefala, 2006). In OECD countries (OECD 2004) there is detected a positive association between the length of time of IT use and students' performance in PISA mathematics tests. But in a meta-analysis investigating different methods for teaching in the secondary-algebra classroom, Haas (2005) found no effects (d=0.07) of technology-aided instruction using computer software applications and/or hand-held calculators. However, in an earlier meta-study Hembree and Dessart (1986) found that the *pedagogical* use of calculators in precollege mathematics education improved student's basic skills both in completing exercises and problem solving.

As a consequence The US-National Mathematics Advisory Panel (2008, p. xxiv) concludes that the available research is insufficient for identifying the factors that influence the effectiveness of instructional software under conventional circumstances. One of the arguments often met is that IT impacts on competency development - like team work and higher order thinking skills - are activities that are not yet recognized by the education systems with ways of assessing them. Since our study is based on results from a competition, and hence not a traditional evaluation system, this kind of approach may provide new input to the ongoing discussions.

The KappAbel contest

The choice of the "best practice" classes was done of the basis of the national Scandinavian KappAbel mathematics contest for ninth grades (see: http://www.kappabel.com/index_eng.html). The overall aims of the competition are (1) to influence the students' beliefs and attitudes towards mathematics and (2) to influence the development of school mathematics. The name "KappAbel" is first and foremost about being capable (Norwegian: kapabel). In addition, the name is meant to honour the Norwegian mathematician Niels Henrik Abel (1802-1829).

The KappAbel contest rewards problem solving of relatively open-ended tasks and other type of skills than just routine exercises in mathematics. It is based on the following ideas:

1. The whole class collaborates in two introduction rounds and hands in joint solutions
2. The class is doing a project work with a given theme.
3. Each class is represented by a team of four students, two boys and two girls, in the national semi-final and final.

First the participating classes within each of the 19 Norwegian counties (fylker) compete in the two introductory rounds of the competition, which are held locally. In Norway one class from each county qualifies. The local winners succeed onto the semi-final, and prepare a project that will make 1/3 of the ruling in the semi-final. The topic of the project is given by KappAbel. In 2007/08 the theme was "Mathematics and animals" When they meet for the semi-final, the students present the results of their project work in a report, a log book and at an exhibition. The three best classes meet the next day for the national final.

Around 20 % of the Norwegian schools participate in KappAbel. In the school year 2007/08 574 classes from 243 schools joined, which means that more than 10 000 students were involved in round 1 and 2. These first parts of the contest are based on teamwork performance by the whole class, not by individual students.

In the present study we address these research questions:

1. What are the common features found between teachers who belong to a school who repeatedly achieve a high learning performance in mathematics?
2. What characterizes their pedagogical praxis?

The study is both a description of some best praxis in the field of mathematics education, and the analysis of the "active ingredients" that make them be of such excellence.

2. Methodology and data

Research design

Case-control studies provide a research method for investigating factors that may cause or prevent success (Schlesselman, 1982). Basically the method involves the comparison of cases with a group of controls. The comparison is aimed at discovering factors that may differ in the two groups and explain occurrence of success. In the KappAbel study, we apply a comparative design from stratified data, with strata defined by the 19 Norwegian counties (fylker). The data-set was increased by starting with two cases in each of the counties. In each county the sample of two cases were matched with controls from the same socioeconomic background. The data must be considered as a strategic sample, based on overall coverage of the country.

Cases comprised the local record of winning classes in the year 2008 from each of the 19 counties. Cases belonging to schools with no previous top results were excluded, and hence in each county we selected the two best classes from schools with repeated top results. From each county we define a top class in 2008 as a "best practice class in mathematics" if its school earlier has been among the 5 best in the local KappAbel competition (it started in year 2000). This is to exclude classes where the result is dominated by one or two very clever students. If the school has not been on the top 5 list earlier, we test the next one by the same procedure, and so on.

Controls should be comparable and similar to cases and were obtained by the principle of matched sampling. Matching involved the pairing of one control to each case by selecting a near neighbour to each school above that enrol students with the same socioeconomic status (SES). Schools with top results in the previous three years of the local competition were excluded. By this sampling strategy it is possible to eliminate some of the effects from social and geographical variables. In most circumstances, a matched design results in a modest improvement in efficiency in detecting an association (Schlesselman, 1982 p. 116).

Design of the questionnaire

The actual approach of the study aims to measure what kind of activity and output that leads to skills in mathematics based on self-reported perceptions of the teacher. The questionnaire had a descriptively purposes where one wants to describe teachers activity in education. It also has an analytical objective where one looks for relationships between teachers' backgrounds, educational qualifications, pedagogical practices and how IT is a priority.

Thematically, the questionnaire can be grouped into five different sets of questions:

1. Teachers' education, teaching experience, and IT skills
2. Teachers' prioritization of educational activities
3. The uses of software in use at the math education, and how
4. Teachers' opinion about IT in relation to pupils' learning performance
5. The teachers' attitudes to mathematics and its educational goals

The questionnaire contained closed questions and opens a possibility to write comments at the end. A part of the survey maps the affective conditions, perceptions and attitudes to mathematics and mathematics teaching. In this context it is used Likert-scales where teachers are asked to take a position on questions and statements by checking one of six options.

A 98 item self-report questionnaire was designed to explore background variables and perceptions of competence. The study and its purpose were described on a separate page in the questionnaire according to standards prescribed by the Norwegian Data Inspectorate, and the questionnaire was sent to the mathematics teacher in the best practice classes. These teachers also had helpful local knowledge about a matching neighbour school to go on with. The same questionnaire was used with the KappAbel teacher, and the mathematics teacher at the comparing school. For their contribution, the teacher was offered a book as a personal gift.

The sample (N=38+38) included 4 classes from grade 9 from each of the 19 Norwegian cantons. It was easy to establish contact with the 38 case-teachers, since this was an interview with “winners”. The subsequent 38 teachers had to be followed up by SMS and telephone. Three of them refused to answer the questionnaire, and a substitute in the same neighborhood had to be selected. The final response rate was 100% (N=76). Forty-two percent of the teachers were women both in the Kappabel group and the control group. Data related to scores in round 1 and 2 of the Kappabel competitions were also collected. For the Kappabel group these were known for each class, but for the control group we had to use the mean value obtained in the canton where the school belonged.

Statistical analyses

The questionnaire contains a number of individual variables. Some of these are meant to function separately, but many of them are part of a collective variable, and these aggregate variables represent values of a *construct*. By this the number of variables in the questionnaire were reduced to more fundamental constructs based on the logical content of the question and reliability testing, with the number of items from 4 to 6, and Cronbach alpha reliability scores value of 0.60 or above to assess statistical quality of the construct.

The matching of data should be accompanied by a statistical analysis that corresponds to the matched design. The data samples from the winning class and its appropriate social neighbour were compared by a paired test (paired Wilcoxon nonparametric test). By this we may infer the difference in use of i.e. IT in a “best practice class in mathematics” compared to the baseline. The null-hypotheses are that there is no difference.

It is also natural to be able to provide a measure of the size differences between two groups. What constitutes a "big" difference for a particular variable depends on how widely spread it is in the material as a whole. A usual way is to define the differences as standardized differences, also called effect size: How big the difference is compared to a standard deviation (King and Minium, 2008 p. 258).[3] We calculated the effect size and report this measure (*d*-value) together with ordinary p-values.

3. Results

The data were collected from matched pairs of schools, paired on the basis of location and SES. Thus, the paired Wilcoxon test was applied to compare the means of the variables between the Kappable and the control classes. In Table 2 we report all significant results found for single variables. The research questions were further operasionalized and analyzed

[3] It remains controversial as to whether the correlation between case and control should be taken into account when calculating an effect size estimate for matched pair designs. Some thinks so (e.g., Rosnow & Rosenthal, 1996 and 2009), but others don't (Dunlop, et al. 1996). We calculated the effect size by not taking the correlation into account, and report this measure or *d*-value, together with ordinary p-values. The estimator for effect size is given by the difference between the means of the case- and the control group, divided by the pooled sample standard deviation, and with equal sample size it is found as the square root of the average of the squared sample standard deviation:

$$\hat{d} = \frac{Mean_{Case} - Mean_{Control}}{SD_{pooled}} = \frac{Mean_{Case} - Mean_{Control}}{\sqrt{(SD^2_{Case} + SD^2_{Control})/2}}$$

through five main constructs as also listed in Table 2, with Cronbach alpha reliability scores of 0.60 or above.

As can be seen in Table 2, more teachers in the Kappable classrooms studied mathematics in universities rather than in colleges (M = 1.61 and 1.11, respectively, $p < .0001$). Kappable classes were more often engaged in hypothesis testing than the control ones (M = 3.34 and 2.79, p = .056); more Kappable students are using ICT for research, exploration and calculation than control students (M = 4.13 and 3.50, respectively, p = .01); Kappable classes use portfolio for reporting progress of students' projects more than the control classes (M = 2.71 and 1.71, respectively, p = 002); students in Kappable classes, more than their peers in the control classes, are encouraged to evaluate their strategies of solving math problems (M = 4.55 and 4.11, respectively, p = .038); and more Kappable students are using spreadsheets than students in the control classes (M= 4.13 and 3.79, respectively, p = .036).

Importantly, more Kappable teachers adhered to a reason-based understanding (M = 4.03 vs. 3.70, p =.024) while more of the control teachers adhered to a rule-based instrumental understanding approach to teaching mathematics.

Comparison of the means of specific questionnaire items pertaining to the use of a variety of IT tools, yielded no significant differences between the groups, expect for spreadsheets and portfolios and overall IT use. The general and social use of Internet does not show any difference in the material. The Kappabel *teachers* themselves, however, use digital mathematical Internet resources somewhat more than their peers (M=3.68 vs. M=3.24, p =.062). More of the control teachers adhered to use IT to stimulate students to figure out ways to solve problems without help from the teacher, and to student collaboration via Internet. Last, Kappable and control classes did not differ significantly from each other on such variables as use of calculators, engagement is projects and – most importantly – in their math grades as measured by the traditional mid-term evaluation.

Table 2: Wilcoxon paired test between Kappable (N=38) and Control Classes (N =38). Column three shows the accompanying d-values (effect size). A positive d-value indicates a higher score in the Kappabel group than in the control group, and a negative d-value indicates the opposite.

VARIABLE	**Statistical test**	**Effect size**
Organizing teaching:	ns	
The teacher:		
Teaching experience, mathematics	0.051	0.52
Mathematics education, credit points	0.044*	0.34
Formal degree (Teachers College/ Univ. Bachelor/ Univ. Master)	0.003**	0.82
College/ University	<0.0001**	1.21
Content and activities:		
Students plan and test hypotheses	0.056	0.56
Challenge students to evaluate and substantiate their strategies	0.038*	0.53
Use of digital tools for exploration and computation	0.010*	0.71
Reform based ©	0.29	0.24
Traditional ©	0.54	-0.07
Reason oriented ©	0.024*	0.53
Rule oriented ©	0.83	-0.13

IT usage:		
Spreadsheet	0.036*	0.45
Digital portfolio to hand in exercises	0.002**	0.77
Overall use of IT ©	0.025*	0.50
Use of IT stimulate students to figure out ways to solve problems without help from the teacher	0.016*	-0.46
Students can to a greater extent help each other through collaborating over the Internet	0.031*	-0.44

© = Constructs combining single variables on the basis of the logical content of the variables and reliability control.
* $p < .05$; ** $p < .01$

These analyses reveal that teachers' level of education makes a significant difference when the success in carrying out math projects in problem solving is the learning criterion, coupled by the flagship of the practiced open-ended pedagogy: Exploration and hypothesis testing.

4. Discussion and conclusion

The intention of the present study was to look at the diversity between school classes that were found to be doing very well in applied math competition and comparable average achievement classes. Information was collected from teachers of the two groups of classes.

The most important influence on individual differences in teacher effectiveness is teachers' general intellectual ability as documented by the formal academic competence, followed by experience and subject matter and content knowledge. The initial academic competence of the teacher is the best predictor for good results. In our material subject matter knowledge of the teacher is also observed to be significantly related to student achievement, a result that is consistent with Hill, Rowan and Ball (2005) and Falch and Naper (2008). More teachers in the Kappable classes have a university rather than college grade which means that they have been exposed to a full load of math studies and are likely to have a better mastery of that subject, and the teaching is reason-based. Moreover, these teachers view IT as a tool for exploration at the expense of using it as a teaching device. Our results show that a subject specific tool like spreadsheet is more in use in the Kappable, best practice classes, than in the control ones. In other words, it is not IT *per se* but more reasoning-based pedagogical student activity, for which IT is used, that makes the difference.

For the time being, spreadsheet seems to be the only discipline specific digital tool that makes a significant positive effect for lower secondary school mathematics. We may therefore conclude that there are good reasons that spreadsheets should be considered as a useful tool in developing students' fast, accurate, and effortless performance on computation, freeing working memory so that attention can be directed to the more complicated aspects of a problem. Spreadsheets are characterized as a tool that are a free, open and flexible resources that allow for exploratory activities in the mathematics subject and can help promote understanding (Goos et al., 2005; Haspekian, 2005; Fuglestad, 2007; Erfjord & Hundeland, 2007). It seems that the best practice teachers acknowledge this. If other digital tools will have the same qualitative impact within school mathematics is yet to be seen. Dynamic geometry tools are slightly more used among high performance classes, but programs of this type seems not to be so familiar and valuable that reaching the necessary program understanding pays off in mathematical understanding. The impact of IT on mathematics and teaching in general has been heavily dependent on the political objectives related to IT. However, research should not focus on IT alone, but include wider didactic topics such as subject specific innovations and find instruments to capture and detect this kind of sustainable results and processes.

Regarding the overall methodological approach, the research findings in this paper are assumed to be reasonably valid, although here might be some bias in the selection of control classes. A case-control study is in general considered to have some limitations in relation to a regular population based study (Schlesselman, 1982), and our results must be handled with some care. One of the further reservations of the present study is that it is mainly based on teachers' *beliefs* of what is going on in the classroom. The research literature in mathematics education points to an often observed inconsistency between teachers' beliefs as expressed in interviews and questionnaires, and their actual praxis in the classrooms (Thompson, 1992; Raymond, 1997; Beswick, 2005). Since our data collection relates to a particular class at grad 9, this may possibly increase the validity of the questionnaire. But further elaboration and real classroom observations are needed to verify the results.

Teachers have to be encouraged to become active shapers of the reasoning and learning process. This requires a professional environment and culture that allows teachers to do so. Training programs should be more adapted to subject specific needs of teachers that can serve the learning of mathematics.

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